

Příklady na procvičení – Funkce: Goniometrické funkce, rovnice a nerovnice

1. Řešte rovnice v \mathbb{R} :

a) $\sin 3x = 1$

c) $2 \cos^2 x - \cos x - 1 = 0$

e) $2 \sin^2 x - \sin x = 0$

g) $\cos x = -\sqrt{3} \sin x$

i) $\sin 4x = \sin 2x$

b) $\sin(1 - x) = 0$

d) $\operatorname{tg} x + \operatorname{cotg} x - 2 = 0$

f) $4 \cos^3 x = \cos x$

h) $\sin x + \sin 2x = 0$

j) $\cos 4x - 3 \cos 2x - 3 \sin 2x = 0$

2. Řešte nerovnice v \mathbb{R} :

a) $\sin x \geq \frac{1}{2}$

c) $\operatorname{tg} x > \sqrt{3}$

e) $\sin x \geq \cos x$

g) $2 \sin^2 x > 3 \cos x$

b) $\cos x \leq -\frac{\sqrt{3}}{2}$

d) $\operatorname{cotg} x \geq -1$

f) $\cos x \cdot \sin x > 0$

h) $\sin^2 x + 3 \cos x - 3 \leq 0$

3. Nakreslete do jednoho obrázku grafy funkcí:

a) $f_1: y = \sin x$; $f_2: y = 2 \sin x$; $f_3: y = 2 \sin x - 3$

b) $f_1: y = \cos x$; $f_2: y = -0,5 \cos x$; $f_3: y = -0,5 \cos x + 2$

c) $f_1: y = \operatorname{tg} x$; $f_2: y = \operatorname{tg}\left(x + \frac{\pi}{2}\right)$; $f_3: y = \operatorname{tg}\left(x + \frac{\pi}{2}\right) - 3$

d) $f_1: y = \operatorname{cotg} x$; $f_2: y = |\operatorname{cotg} x|$; $f_3: y = |\operatorname{cotg} x| + 2$

Řešení:

1. a) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{1}{6}\pi + \frac{2}{3}k\pi \right\}$

c) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{2}{3}k\pi \right\}$

e) $K = \bigcup_{k \in \mathbb{Z}} \left\{ k\pi; \frac{1}{6}\pi + 2k\pi; \frac{5}{6}\pi + 2k\pi \right\}$

g) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{5}{6}\pi + k\pi \right\}$

i) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{1}{6}\pi + k\pi; \frac{5}{6}\pi + k\pi; k\frac{\pi}{2} \right\}$

2. a) $K = \bigcup_{k \in \mathbb{Z}} \left\langle \frac{\pi}{6} + 2k\pi; \frac{5\pi}{6} + 2k\pi \right\rangle$

c) $K = \bigcup_{k \in \mathbb{Z}} \left\langle \frac{\pi}{3} + k\pi; \frac{\pi}{2} + k\pi \right\rangle$

e) $K = \bigcup_{k \in \mathbb{Z}} \left\langle \frac{\pi}{4} + 2k\pi; \frac{5\pi}{4} + k\pi \right\rangle$

g) $K = \bigcup_{k \in \mathbb{Z}} \left\langle \frac{\pi}{3} + 2k\pi; \frac{5\pi}{3} + 2k\pi \right\rangle$

b) $K = \bigcup_{k \in \mathbb{Z}} \{1 + k\pi\}$

d) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{1}{4}\pi + k\pi \right\}$

f) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{1}{2}\pi + 2k\pi; \frac{1}{3}\pi + k\pi; \frac{2}{3}\pi + k\pi \right\}$

h) $K = \bigcup_{k \in \mathbb{Z}} \left\{ k\pi; \frac{2}{3}\pi + 2k\pi; \frac{4}{3}\pi + 2k\pi \right\}$

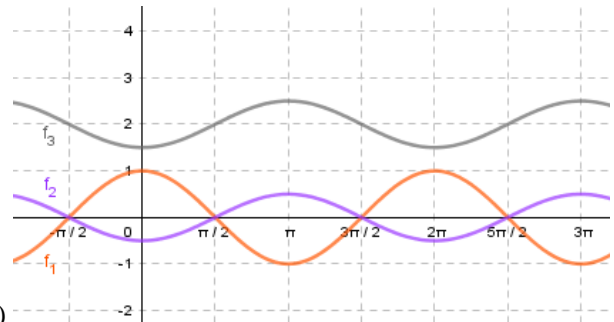
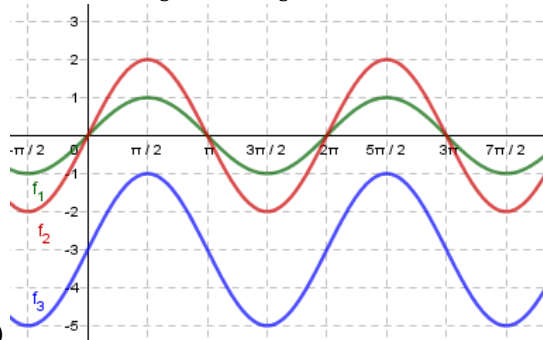
j) $K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{3}{8}\pi + k\frac{\pi}{2} \right\}$

b) $K = \bigcup_{k \in \mathbb{Z}} \left\langle \frac{5\pi}{6} + 2k\pi; \frac{7\pi}{6} + 2k\pi \right\rangle$

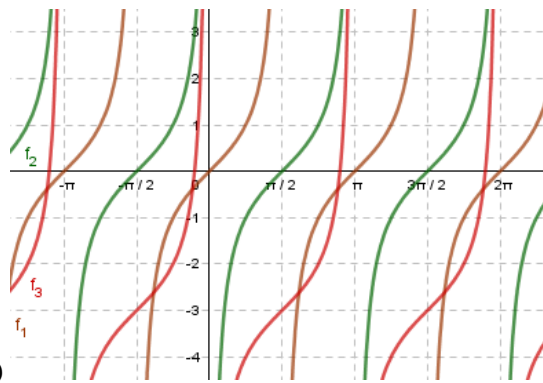
d) $K = \bigcup_{k \in \mathbb{Z}} \left\langle k\pi; \frac{3\pi}{4} + k\pi; \right\rangle$

f) $K = \bigcup_{k \in \mathbb{Z}} \left(0 + 2k\pi; \frac{\pi}{2} + 2k\pi \right) \cup \left(\pi + 2k\pi; \frac{3\pi}{2} + 2k\pi \right)$

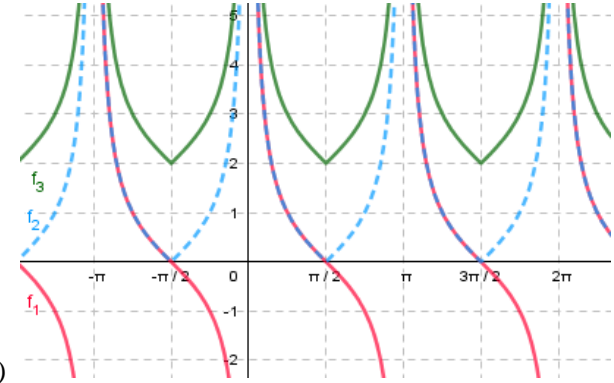
h) $K = \mathbb{R} - \bigcup_{k \in \mathbb{Z}} \left\{ \frac{k\pi}{4} \right\}$



3. a)



b)



c)

d)